A persistent particle ontology for QFT in terms of the Dirac sea

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We show that the Bohmian approach in terms of persisting particles that move on continuous trajectories following a deterministic law can be literally applied to QFT. By means of the Dirac sea model – exemplified in the electron sector of the standard model neglecting radiation – we explain how starting from persisting particles, one is led to standard QFT employing creation and annihilation operators when tracking the dynamics with respect to a reference state, the so-called vacuum. Since on the level of wave functions, both formalisms are mathematically equivalent, this proposal provides for an ontology of QFT that includes a dynamics of individual processes, solves the measurement problem and explains the appearance of creation and annihilation events.

Keywords: Quantum field theory, Dirac sea, particle ontology, Bohmian mechanics, vacuum, particle creation and annihilation

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1 Bohmian mechanics from QM to QFT

Bohmian mechanics (BM) is by now a well-respected proposal for the ontology of non-relativistic quantum mechanis (QM). Any such proposal has to provide a solution to the measurement problem by setting out what happens in nature on the level of individual quantum systems instead of merely making statistical predictions for measurement outcomes. BM meets this requirement by supplementing the wave equation with a guiding equation that yields trajectories for individual particles in three-dimensional space (what is known as the primitive ontology) and that explains the states of macroscopic systems as well as their stability in terms of these trajectories (see Dürr et al. (2013a)).

When it comes to QFT, the requirement for an ontology is the same as in QM: the measurement problem plagues QFT in the same way as QM (see Barrett (2014)); any proposal for its solution has to explain what happens in nature on the level of individual processes instead of merely making statistical predictions. On the one hand, a particle ontology is generally regarded as excluded for QFT: if there is not a definite number of particles that persist, particles cannot be what is fundamental (see e.g. Halvorson and Clifton (2002); Kuhlmann (2010, chapter 8); Ruetsche (2011, chapters 9-11)). On the other hand, it is by no means clear that the ontology of QFT is one of fields, since QFT does not admit fields that have definite values at the points of four-dimensional space-time (see Baker (2009)).

Against this background, our claim is that the Bohmian approach works for QFT in the same way as for QM: as it is a *non sequitur* to take particle trajectories to be ruled out in QM due to the Heisenberg uncertainty relations, so it is a *non sequitur* to take permanent particles moving on definite trajectories according to a deterministic law to be ruled out in QFT due to the statistics of particle creation and annihilation phenomena. In both cases, such an underlying particle ontology is in the position to explain the statistics of measurement outcomes. Our claim therefore is that if the Bohmian approach is a serious contender for the ontology of QM, then it is also a serious contender for the ontology of QFT. We only seek to establish this conditional claim. We are not concerned here with defending the Bohmian one against rival approaches.

Bohm himself in his Bohm (1952), appendix A, already discussed a possible extension of his theory to the electromagnetic field. Indeed there is a field version of Bohmian QFT (see Struyve and Westman (2006, 2007) as well as Valentini (1992), chapter 4). As regards the particle ontology, the first Bohmian QFT can be traced back to Bell (1986) (see Dürr et al. (2005) for a continuum version of Bell's proposal on a lattice; see Struyve (2010) for an overview of all the Bohmian proposals). Bell's proposal grants the entities that figure in the particle-pair creation and annihilation processes of QFT the status of real, fundamental particles, which are, accordingly, created and annihilated at random times and positions. Between these random events, the particles evolve according to a Bohmian law of motion.

We do not adopt this proposal. In the first place, since these entities depend on the choice of an initial reference vacuum state that is not unique (see Fierz and Scharf (1979)), instead of being objects that simply exist, we consider it to be inappropriate to grant them the status of fundamental objects in the ontology. More generally speaking, the Bohmian approach is motivated by finding an underlying ontology that explains the statistics of measurement outcomes. We think that it is worth while to pursue this approach only if one is prepared to go all the way down to an ontology of fundamental objects that are simply there – that is, that do not come into existence out of nothing and that do not disappear into nothing – and that evolve according to a simple, deterministic law (that is, an evolution not interrupted by random jumps).

That is why we take up an idea going back to Dirac (1934): there is no particle creation or annihilation. There are only conditions under which particle motion becomes observable or fails to be so. These conditions are not unique; they may even depend on the state of motion of the observer (as in the Unruh effect). We build an ontology of permanent particles on this idea. In this task we face a shortcoming of the modern formulation of QFT, which is given only in terms

of a scattering theory. The mathematical difficulties involved in the computation of scattering matrix elements are already so severe that in the renormalization group formulation of QFT, the question about a fundamental equation of motion is hardly thematized. In ontology, however, we have to make explicit what the laws are according to which individual processes occur; for what there is in nature are individual events and processes. Techniques developed to calculate measurement outcome statistics do not reveal how these processes evolve from an initial to a final state.

We therefore propose to return to the early attempts of the 1930s to define equations of motion for QFT. In doing so, we will not obtain better predictions of measurement outcomes than standard QFT. The situation is the same as in BM: the guiding equation does not yield better predictions; accordingly, it does not have to be solved to calculate statistical predictions of measurement outcomes. It figures in the explanation of these statistics by answering the question of what is really going on ontologically. Hence, the purpose in returning to Dirac (1934) is not to pursue outdated physics, but to make progress with respect to the ontology of QFT. In modern formulations of QFT, a mathematical description in terms of the Dirac sea, although being canonically equivalent on the level of wave functions, has been abandoned in favour of a more economic description involving particles and anti-particles as well as their creation and annihilation. It is not our intention to reintroduce the Dirac sea as computational device. On the contrary, we will argue that after it served its ontological purpose, it suggests itself to develop computational methods that let go the bulk of the Dirac sea, replacing it by a so-called vacuum state, and only track its excitations. What one gains in formulating the fundamental equations of motion in terms of the Dirac sea is an ontological explanation of the physical processes behind particle and anti-particle creation and annihilation.

In the following, we examine the so-called *Dirac sea model* or *hole theory*, which was the first one to predict the phenomenon of electron-positron pair-creation. Although it is conceived only for the electron sector of the standard model (SM) of QFT, it is applicable to all fermionic matter. As Bell (1986) argued, the commitment to fermions is sufficient to account for the empirical data: they all consist in a spatial arrangement of fermions. Hence, if the Dirac sea model can cover all fermionic matter, this is all that can be reasonably demanded for it to serve as a proposal for the ontology of QFT. Furthermore, this model can be given a rigorous mathematical treatment in certain regimes. It is therefore suitable for probing the compatibility of QFT with an ontology of persistent particles. Colin (2004) and Colin and Struyve (2007) have carried out an investigation in that sense in the physics literature (see also the brief remarks in Bohm et al. (1987, pp. 373-374) and Bohm and Hiley (1993, chapter 12.3)). In the philosophical literature, the Dirac sea model is hitherto largely ignored, apart from a brief assessment in Saunders (1999, pp. 78-79, 86-88).

In the next section, we introduce the Dirac sea model (section 2). Then, we define the state of equilibrium for the particles – that is, a state in which nothing can be observed, by physicists usually referred to as the "vacuum state" (section 3) – and small excitations from equilibrium, which can be observed and which appear as if there were a creation and annihilation of particles. This then is the basis on which we show how the formalism that effectively describes these excitations leads naturally to the standard QFT formalism, which employs creation and annihilation operators. We discuss how, due to the ontology of persistent particles, it is possible to give a clear meaning to these operators as well as to the vacuum state (sections 4 and 5). We conclude with showing how this ontology and dynamics explains the measurement outcomes, arguing that it does for QFT what BM does for QM.

2 The Dirac sea model

The Dirac sea model can be inferred from the SM by imposing the following restrictions:

1. Restriction to the electron sector of the SM;

2. Restriction to direct electrodynamic interaction and neglect of radiation;

3. Modelling interaction with all other fermion sectors of the SM only effectively by a time-dependent "external" interaction.

The resulting model is simple enough to enable an unobscured discussion, but has sufficient structure to describe the phenomenon of electron-positron pair-creation. It therefore serves our purpose well. In posing these restrictions, we assume that the particles can be divided into two groups: the first group – below labelled by $1, \ldots, N$ – are electrons, while the rest are other fermions. Nonetheless, we emphasize again that the restriction to electrons is arbitrary, since the Dirac sea model and hence our argument is also applicable to the other fermion sectors and the other types of interaction in the SM.

Electrons in direct electrodynamic interaction exclusively repel each other. If this were the only interaction, the spatial extension of this cloud of N electrons would inevitably grow larger and larger without any bound. This is an artifact of restrictions 1-3, as we model only the electron sector directly (though we allow an indirect, effective interaction to other fermion sectors). To avoid such unphysical behaviour we stipulate further that, due to the motion of all the other particles and their interaction by means of an "external interaction" (which then includes also attractive interactions, even gravitation, although the latter is not described by the SM), the spatial extension will always be bounded (which might also be suggested by general relativity and supported by cosmological evidence). Furthermore, we assume that this "external" interaction is reasonably well behaved in the following sense: neither will it dampen the motion of the electrons to such an extent that all electron motion comes to a rest, nor will it drive the electron velocities arbitrarily close to the speed of light. Thus, there is no infinite energy transfer. The idea behind this assumption is that motion should be somewhat conserved among all fermion sectors. Neither does motion arise from nothing nor does it cease to exist, it only varies over the individual particles.

Posing these two additional restrictions allows us to avoid the discussion of mathematical problems – notably the infamous ultraviolet / infrared divergences –, which are the obstacle in finding a well-defined equation of motion in QFT. Put mathematically, we assume that the initial data and the "external" interaction in our theory are such that:

4. The spatial extension of the universe is restricted to a finite volume Γ in 3d-space $\mathbb{R}^3$.

5. The electron momenta are restricted to be lower than some finite ultraviolet cut-off Λ.

In a yet to be found well-defined SM of QFT, the above assumptions must not be imposed *a priori*, but have to come out *a posteriori*. In particular, the gained mathematical well-definedness comes at the cost of a violation of Lorentz invariance and the introduction of seemingly arbitrary parameters. Accordingly, the main objective of modern renormalization theory is the removal of these unwanted cut-offs. As yet, however, a removal is only achieved order by order in the informal expansions of scattering matrix elements. When it comes to an equation of motion, the introduction of cut-offs is still standard in QFT. Nevertheless, it is the hope amongst physicists that provided the cut-offs are large enough, the effects on the dynamics are only small in certain regimes of interest. In fact, given the cut-offs, the formulation presented here is equivalent to the textbook formulation on the level of wave functions. By introducing another cut-off in the respective Bose fields, it is furthermore possible to include the full electroweak interaction including radiation effects and remove assumptions 1-3 altogether (see Colin and Struyve (2007)). In general, the Dirac sea model applies to any fermion sector and potentially any interaction between those fermions (except maybe gravitation). In short, the above assumptions are no objectionable limitations of our endeavour. They rather underline the general point that the rationale of the Bohmian approach is not to obtain better physics, but an ontology for the existing physics.

The Dirac sea model was originally proposed by Dirac (1934, 1947) in Hartree-Fock approximation in terms of density matrices. Without this approximation it was later included in QFT by means of a second quantization procedure of the one-particle Dirac equation yielding the so-called Dirac field operators on Fock space. We discuss the Dirac sea model in terms of a straightforward quantum mechanical N-particle wave equation. For finite Γ and Λ, this formulation is equivalent to the textbook version (see section 4). Additionally, we conceive the Dirac sea model in terms of BM.

The Bohmian velocity field v_t guiding the motion of N particles can be given in the form

$$v_t(X) = c\left(j_t^{(k)}(X)/\rho_t(X)\right)_{k=1,\dots,N}, \tag{1}$$

$$\rho_t(X) = \Psi_t(X)^*\Psi_t(X), \quad j_t^{(k)}(X) = \Psi_t(X)^* 1^{\otimes(k-1)} \otimes \alpha \otimes 1^{\otimes(N-k)}\Psi_t(X), \tag{2}$$

where $X = (x_1, \dots, x_N) \in \Gamma^N$, and ρ_t and $j_t^{(k)}$ for $k = 1, \dots, N$ are the probability density and currents generated by a wave function Ψ_t and c stands for the speed of light. Here, Ψ_t denotes an anti-symmetric, square-integrable, N-particle spinor-valued function on configuration space $\Gamma \subset \mathbb{R}^{3N}$, in short $\Psi_t \in \mathcal{H}^{\wedge N}$ for $\mathcal{H} = L^2(\Gamma, \mathbb{C}^4)$, that solves the wave equation

$$\begin{aligned} i\hbar\partial_t\Psi_t(x_1, \dots, x_N) &= H^N\Psi_t(x_1, \dots, x_N), \\ H^N &= \sum_{k=1}^{N}\left(H_k^0(x_k) + V_k(t, x_k) + H_k^I(x_k)\right). \end{aligned} \tag{3}$$

The Hamiltonian H^N is made of the free Dirac Hamiltonian $H_0^k(x)$; the external influences $V_k(x)$ are given by

$$\begin{aligned} H_k^0(x) &= 1^{\otimes(k-1)} \otimes H^0(x) \otimes 1^{\otimes(N-k)}, & H^0(x) &= -ic\alpha \cdot \nabla_x + \beta mc^2, \\ V_k(t, x) &= 1^{\otimes(k-1)} \otimes V(t, x) \otimes 1^{\otimes(N-k)}, & &\text{for some external potential } V(t, x), \end{aligned} \tag{4}$$

where m stands for the electron mass and $\otimes$ denotes the tensor product (note that Ψ is spinor-valued). Furthermore, β and the components of the vector $\alpha = (\alpha_1, \alpha_2, \alpha_3)$ are $\mathbb{C}^{4\times4}$ matrices that fulfill the anti-commutator relations $\{\alpha_j, \alpha_k\} = 0 = \{\beta, \alpha_j\}$ and $\beta^2 = 1 = \alpha_j^2$ for $j \neq k$, $j, k \in \{1, 2, 3\}$. Moreover, H_I denotes the Hamiltonian modelling the interaction between the particles. Since we neglect radiation, the interaction is mediated directly by

$$H_k^I(x) = \frac{1}{2}\sum_{j\neq k} U(x - x_j), \tag{5}$$

where the electric interaction between the N electrons can be taken to be the Coulomb potential $U(x) = \frac{e^2}{4\pi\epsilon_0}|x|^{-1}$. Here, ϵ_0 is the electric constant and $e < 0$ the charge of an electron. Note that H_k^I depends also on x_j, which is suppressed in our notation. Finally, the composite, effective interaction of all particles on electron k is given through a time-dependent potential $V_k(t, x)$, stemming for example from the Coulomb field of a present ion, etc.

In sum, the velocity field (1) and the wave equation (3) define what we call the Dirac sea model. Given an initial configuration Q_0 and wave function Ψ_0, the corresponding solutions of these equations yield a unique trajectory of configurations of the N electrons parametrized by time. In principle, there is nothing more to say about the ontology: there are N persistent electrons that evolve according to a deterministic law of motion. However, we face the following situation:

(I) Generically, we do not have complete knowledge about the initial data (Q_0, Ψ_0).

(II) Even if we did, if N is large (which is the interesting case, as we assume large cut-offs), it

is in general neither analytically nor numerically feasible to compute solutions Ψ_t of the wave equation (3).

Problem (I) is generic to any theory that applies to the universe as a whole. Predictions about subsystems have to be inferred from a detailed statistical analysis of what is to be expected in most situations with respect to a meaningful measure. In BM, this is made possible by the special form of the velocity law (1), which ensures that if the initial data Q_0 is distributed at random according to $|\Psi_0(X)|^2 d^{3N}X$, configuration Q_t is distributed according to $|\Psi_t(X)|^2 d^{3N}X$. This feature is known as equivariance. The statistical analysis of subsystems has been carried out in Dürr et al. (2013a, chapter 2) for BM. This analysis applies to our setting as well. The bottom line is a proof that Born's rule holds true: if the effective wave function of the subsystem is given by $\phi_t(x_1, \ldots, x_n)$ for $n \leq N$, then the distribution of the subconfiguration of particles $q(t) = (q_1(t), \ldots, q_n(t))$ is given by $|\phi_t(x)|^2 d^n x$.

Problem (II) stems from the fact that the pair potential U strongly entangles all tensor components of the wave function Ψ_t during the time evolution. Even a perfect initial anti-symmetric product state will therefore immediately lose its product structure due to (3). The complexity of this entanglement increases exponentially with N. Consequently, one must hope to find special non-trivial situations in which the very complicated N-particle dynamics becomes simple enough so that it can be approximated by studying the motion of a few particles only.

The original motivation for Dirac's hole theory was not the complexity of its solutions, but stemmed from the attempt to make sense of Dirac's equation describing only one particle (see Saunders (1991) for the context of Dirac's theory). One quickly realized that the solutions showed strange behaviour – for instance the *Zitterbewegung* (Schrödinger (1930)) and *Klein's paradox* (Klein (1929)) –, which made Dirac's equation hard to interpret physically. The mathematical reason for this strange behaviour is the presence of a negative energy spectrum of the free Dirac Hamiltonian $H^0(x)$ as given in (4). Therefore, the starting point of Dirac's hole theory was to somehow suppress this negative spectrum. As the electrons are fermions, this can be accomplished by filling all the negative energy states with a sea of particles – and, granted the cut-offs 4-5, thus ending up with an N-particle wave equation as above. The Pauli exclusion principle then prevents the wave function of any additional electron from growing negative energy components, because all negative energy states are already occupied. The consequence is of course that one ends up with the same complexity problem (II) as well, albeit for a different reason.

Our starting point is different. We seek a theory about the universe as a whole, based on an ontology of permanent particles. We bring in Dirac's hole theory as a suitable means to implement that ontology. This model together with its ontology is spelled out concisely in this and the preceding section. However, even though we only treat a simplified model of that theory and only describe electrons while modeling the rest of the particle interactions effectively, it is natural to start with not only one but all the electrons of the universe. The model can thus only be considered as a serious contender, if it is also possible to analyze it mathematically despite of the complexity problem (II), which is why we anyhow have to cope with a large number of electrons N. This is the content of the following sections: our aim is to show how on the basis of this model we can explain the statistical predictions of standard QFT.

Before doing so, let us compare our approach with the discussion of the Dirac sea model in a Bohmian framework by Colin and Struyve (2007). They also assume a finite fermion number; they are committed to positions for fermions for both positive and negative energy particles, whereas there are no bosons in the ontology. Anti-particles are defined as holes in the sea of the negative energy particles. Whereas Colin and Struyve (2007) define the fermion number as the total number of particles minus the number of negative energy particles plus an infinite constant, we are committed to N particles that de facto coincide with the fermions. Nonetheless, even though in our model there is no ontologically significant difference between positive and negative energy particles, one may say with the usual jargon that the fermion number can be

defined as the number of positive energy particles plus the number of negative energy particles, which remains constant. Colin and Struyve (2007) propose an equation of motion for fermions that defines the vector velocity field for configurations of particles, which is dependent on the wave function of the system according to the usual Bohmian recipe; the expectation value of the fermion number density is related to the position density: intuitively, the number of fermions in a given region of space corresponds to the fermion positions in that region. This model is regularized with the introduction of ultraviolet momentum cut-offs and finite space; these constraints ensure that it is a mathematically well defined theory. Equivariance guarantees that the empirical predictions of the SM with cut-offs are reproduced, exactly as in our case.

We emphasize again that the SM only permits to remove the introduced cut-offs when computing perturbative corrections of scattering amplitudes with respect to non-interacting QFT, while so far the respective dynamics of the SM also becomes ill-defined with any attempt to remove the cut-offs. Since both the theories of Colin and Struyve (2007) and the one developed in this paper are concerned with the dynamics and not with scattering theory, they can only be compared to the SM with cut-offs – until eventually a formulation of the equations of motion of the SM without cut-offs is found.

3 Equilibrium states and the vacuum

How can we tackle problem (II), that is, find manageable approximations of the in general very complicated dynamics of the Dirac sea model? Clearly, this will be possible only in special situations, that is, for a certain class of initial quantum states. Furthermore, such an approximation cannot be carried out without coarse graining the level of detail of the information that is to be inferred about the system.

Consider, as an analogy, a classical gas of N particles confined to the volume Γ. Although it is in principle possible to infer the actual motion of the individual particles by solving Newton's law of motion, for large N, this would be a hopeless endeavour and for many practical purposes unnecessary. For instance, even in equilibrium, the *microscopic*, actual Newtonian motion of the particles may be very intricate. However, effectively the net result is that, *macroscopically*, the gas density is almost constant with the variation around this constant density being small. The smallness requirement depends on the practical purpose as it determines which effects will be visible and which ones will drown in the fluctuations. Hence, it defines what is meant by *macroscopic*. In the same vein, one can introduce further macroscopic parameters like volume, pressure and temperature. For most engineering purposes their relationship constitutes a satisfactory description of equilibrated gases, that is, the theory of thermodynamics. Such a coarse grained description is not restricted to equilibrium states only. For example, in certain regimes it is possible to describe the mediation of a perturbation created in the gas by an external influence in terms of an effective equation for pressure (sound) waves that excite the initially equilibrated gas, without knowledge of the actual microscopic Newtonian motion of the individual particles.

Using this classical example as a guide, let us pursue the idea of describing complex dynamics in terms of small deviations from an equilibrium whose time evolution can be understood in more simple terms. At first, we only consider $V(t, x_k)$ in equation (3) to be zero. That is to say, we stipulate that there are no external influences: the motion of the electrons can be represented in terms of (1) and (3), that is, subject to the Fermi repulsion (due to the anti-symmetry of Ψ_t) and the Coulomb repulsion only. In analogy to the classical gas, the simple approximate solutions inferred in the following play the role of the equilibrium states. In the next section, we then consider the case $V(t, x_k) \neq 0$, which leads to excitations of the equilibrium – the analogue to the pressure waves in the classical example.

As already indicated, the term creating problem (II) is the Coulomb pair-interactions encoded in H_k^I given by (5). We therefore define a class of quantum states for which an approximation

is possible in which this term effectively vanishes and call it the class of equilibrium states. This class shall consist of all states $\Psi \in \mathcal{H}^N$ that fulfill the following two conditions: (a) The quantum mechanical expectation of the interaction operator H_k^I is approximately constant, that is, there is a constant E^I such that

$$E^I \approx \langle \Psi, H_k^I(x)\Psi\rangle = \Big\langle \Psi, \frac{1}{2}\sum_{j\neq k} U(x-x_j)\Psi \Big\rangle \qquad \text{for all } x \in \mathbb{R}^3 \text{ and } k = 1,\dots,N; \tag{6}$$

(b) the fluctuation around this constant expectation value is sufficiently well behaved.

In view of the weak law of large numbers, these conditions (a) and (b) can only be met for sufficiently large N. As the theory so far is meant to apply to the total number of electrons in the universe, N is naturally large. The exact sense of "approximatively" and "sufficiently" depends on the practical purpose: conditions (a) and (b) are there to make sure that the solutions to the fundamental equation of motion (3) for $V(t, x_k) = 0$, given an initial state Ψ_0 in this class of equilibrium states, can for all practical purposes be sufficiently well approximated – e.g. in the sense of reduced density matrices – by a solution to the much simpler effective equation of motion

$$i\hbar\partial_t \Psi_t^{\approx}(x_1,\dots,x_N) = \sum_{k=1}^{N}\Big(H_k^0(x_k) + E^I\Big)\,\Psi_t^{\approx}(x_1,\dots,x_N), \tag{7}$$

replacing the complicated interaction H_k^I by the constant E^I.

Regarding the precise mathematical requirements of conditions (a) and (b) and the precise sense of the approximation we are purposely vague, since the exact behaviour of the fluctuation needed to carry out the rigorous mathematics is not entirely settled in the fermionic case (unlike the bosonic case, where for example Gross-Pitaevski and mean-field approximations can be rigorously derived). If an initial state Ψ_0 fulfills (a) and (b) in a sufficiently strong sense, the corresponding fully interacting time evolution (3) is close to the non-interacting one (7) as the errors which depend on the accumulated fluctuations (b) around the expectation value (a) can be controlled – at least in the sense of reduced density matrices and for large enough N and gas densities. However, while fermionic N-particle states that fulfill (a) are known (e.g., the non-interacting fermionic ground state, see (10) below), it is unknown whether there are ones that fulfill also condition (b) in a sufficiently strong sense. In fact, it is conjectured that (b) might be too strong and that demanding sufficiently small fluctuations for only those particles with momenta below some threshold might already suffice. This would mean that condition (b) could even be weakened and we would still be able to show the closeness of (3) to the approximate time evolution (7) for such states. The exact notion is however irrelevant to our discussion. The important point is that (7) does not have to be assumed, which would be highly questionable for an interacting Fermi gas, but can be derived from much simpler and plausible conditions such as (a) and (b).

Before considering a pertinent example of such an equilibrium state, let us put the motivation for focussing on the class of equilibrium states in other terms. Assuming that there are only electrons subject to Fermi and Coulomb repulsion, the ground state Ψ^{gs} of such an electron gas is expected to be one in which for almost all initial Q_0 with respect to the relevant measure $|\Psi^{\mathrm{gs}}(X)|^2 d^{3N}X$, the electrons are very homogeneously distributed. Then, if the measure $|\Psi^{\mathrm{gs}}(Q)|^2 d^{3N}X$ gives rise to a homogeneous distribution, our defining condition (6) of the class of equilibrium states is a consequence, and the net effect of H_k^I is a constant potential, canceling out all interactions: on the level of wave functions, the electrons in such a state effectively do not "take notice" of each others presence. Effectively, they move as if they were in a vacuum. Hence, the dynamics generated by (3) is very simple, and, within the bounds of the approximation, it leaves the class of equilibrium states invariant.

A natural representative for a state in the equilibrium class would be the actual ground

state Ψ^{gs} of the interacting system. Its mathematical structure, however, is extremely complicated and to date not accessible due to the discussed entanglement induced by the H_k^I terms. Consequently, we have to find a simpler candidate that replaces Ψ^{gs} in the sense of the above approximation. Physicists usually choose the ground state $\Psi_{\approx}^{\mathrm{gs}}$ of the corresponding approximate equation (7) given by

$$\sum_{k=1}^{N} H_k^0(x_k)\, \Psi_{\approx}^{\mathrm{gs}} = E_{\approx}^{\mathrm{gs}}\, \Psi_{\approx}^{\mathrm{gs}}, \tag{8}$$

where $E_{\approx}^{\mathrm{gs}}$ is the lowest eigenvalue. Since the $H_k^0(x_k)$ commute pairwise, $\Psi_{\approx}^{\mathrm{gs}}$ can be found by studying the spectrum of $H^0(x)$ only, which is well-known and given by $\pm E(p) = \sqrt{p^2+m^2}$ for $p \in \mathcal{P}_\Lambda = \{k = (k_1,k_2,k_3)\,|\,|k| \leq \Lambda, k_i = 2\pi/Ln_i, n_i \in \mathbb{Z}, i = 1,2,3\}$. Thus, the Hilbert space $\mathcal{H}$ splits naturally into the one spanned by the positive energy eigenstates $\mathcal{H}^+$ and the negative ones $\mathcal{H}^-$, respectively. The ground state $\Psi_{\approx}^{\mathrm{gs}}$ is then given by the antisymmetric product state

$$\Psi_{\approx}^{\mathrm{gs}} = \varphi_1 \wedge \varphi_2 \wedge \ldots \wedge \varphi_N \tag{9}$$

for φ_n denoting any enumeration of the N lowest eigenstates. Among all such N-dependent ground states Ψ^{gs}, the one for $N = 2|\mathcal{P}_\Lambda|$, in which the total number of particles equals twice the number of admissible momenta (as per $p \in \mathcal{P}_\Lambda$ there are two spin values), is distinguished by having the lowest possible value $E_{\approx}^{\mathrm{gs}}$. Because of this fact, one usually considers the case of a sea of $N = 2|\mathcal{P}_\Lambda|$ many particles, which in physics is referred to as the *vacuum*. Note that this is merely a convenient mathematical idealization (and most probably also not exactly fulfilled in nature, as we seem to see more matter than anti-matter). Furthermore, due to the antisymmetry, one may also choose $(\varphi_n)_{n=1,\ldots,N}$ to be any other orthonormal basis of the subspace $\mathcal{H}^- \subset \mathcal{H}$, which changes the definition in (9) at most by a constant phase factor. As a short-hand notation we will employ $\Omega = \Psi_{\approx}^{\mathrm{gs}}$.

In BM, all there is to the particles comes down to their position in space only and the evolution of this position as given by a guiding equation – in our case equation (1). In other words, in BM, particles are primitive objects in the sense that they only have a position in space. All the other parameters including mass, charge, spin, etc. are not additional elements of the ontology characterizing the particles, but dynamical parameters employed to describe the evolution of the particle positions. Consequently, following BM, particles are not electrons *per se*, but some particles are classified as electrons because they move in a certain manner, namely electronwise so to speak (see Dickson (2000) and Esfeld et al. (2016)). Hence, the appearance of negative energy values in the formalism poses no problem for the Bohmian ontology: an energy value – be it positive or negative – is nothing but a dynamical parameter capturing a particular form of particle motion. The same goes for positrons and anti-matter in general: all these are primitive particles moving in a certain manner. Consequently, the fact that in the vacuum state all the negative energy states are occupied on the level of the wave functions does not imply that the *physical space* is entirely filled with particles. Even in the vacuum state there are finitely many particles that have non-zero distances between them.

It can now be checked directly that for $\Psi = \Omega$ in the equilibrium condition (6), one gets

$$(6) = \langle \Omega, H_k^I(x)\Omega\rangle = \frac{1}{2}(N-1)\frac{1}{N}\sum_{n=1}^{N}\int U(x-y)|\varphi_n|^2(y)\, d^3y, \tag{10}$$

which is constant (note that $|\varphi_j| = 1/|\Gamma|^{1/2}$). This agrees with Dirac's heuristic picture that, in an equilibrium state, the electrons are uniformly distributed as, according to Born's rule, the right-hand side of (10) can be interpreted as expectation value of $\frac{1}{2}\sum_{j\neq k} U(x-x_j)$ for $(N-1)$ uniformly distributed random variables x_j. Beside this, there are unfortunately only

few rigorous results on the quality of the approximation of (7) to (3), that is, whether Ω fulfills also condition (b) above to qualify as a state in equilibrium. The indication that it holds in certain physically interesting regimes comes mainly from the overwhelming accuracy achieved by predictions obtained using (7) whose justification requires conditions (a) and (b) as *a priori* assumptions.

In sum, we have defined a class of initial quantum states, namely the equilibrium class, that allows to solve the in general very complicated dynamics (3) approximately by the much simpler dynamics (7). Furthermore, we have found a particularly simple representative Ω of this class. Solving the approximate equation (7) for the initial value $\Psi_0^{\approx} = \Omega$ yields the explicit and particular simple evolution $\Psi_t^{\approx} = \Omega_t$ for

$$\Omega_t = e^{-i(E_{\approx}^{\mathrm{gs}} + NE^I)t}\Omega, \tag{11}$$

which can now be taken as approximate solution Ψ_t to equation (3) for the same initial value $\Psi_0 = \Omega$. Such a state only gives rise to a trivial dynamics of a sea of N electrons in which none of them "takes notice" of the rest as if they were in a "vacuum" – hence the name *vacuum state*. Nevertheless, this state Ω will provide the basis for studying more interesting dynamics in the next section.

Finally, it has to be emphasized that even with Ψ_t and Ω_t being close for $\Psi_0 = \Omega$ in the sense of reduced density matrices, this provides little information about the closeness of the velocity fields (1) generated by Ψ_t and $\Psi_t^{\approx}$, respectively. In general, the respective velocity fields and the corresponding trajectories differ. However, the results of their statistical analysis agree due to the approximate agreement of the reduced density matrices. In other words, the price that one has to pay to overcome the complexity problem (II) above by means of an approximation in terms of solvable equations is that one has to abandon the hope of obtaining a calculation of actual trajectories in favour of a statistical analysis. This is, however, the same situation as in the example of the classical gas discussed at the beginning of this section. Furthermore, it is the same situation as in BM: there also is no point in calculating individual trajectories, since tiny deviations in the initial configuration will lead to large deviations of the resulting Bohmian trajectories. Consequently, our knowledge of subsystems of the universe is limited to what can be obtained from Born's rule, as proven by Dürr et al. (2013a, chapter 2).

4 Excitations of the vacuum and the Fock space formalism

More interesting dynamics will take place if we allow $V(t, x_k)$ to be non-zero, thus including external influences. Under the action of $V(t, x_k)$, an initial vacuum state Ω, as defined in (9), may evolve into an excited state, as for instance

$$\Phi = \chi \wedge \varphi_2 \wedge \ldots \wedge \varphi_N, \tag{12}$$

where the element φ_1 of the orthonormal basis of $\mathcal{H}^-$ is replaced by a wave function $\chi \in \mathcal{H}^+$. For the sake of simplicity, let us assume that after the transition from Ω to Φ, the external influence vanishes again. As in the preceding section we strive to find an economic effective description of the time evolution of Φ to a Φ_t, $t > 0$ that fulfills the complicated dynamics (3), that is,

$$i\partial_t \Phi_t = H^N \Phi_t, \tag{13}$$

where H^N is the N-particle Hamiltonian defined in (3).

In analogy to the pressure waves in the classical gas example mentioned in the previous section, the leading idea is to describe the complex evolution of Φ_t by the evolution of the excitation only as compared to the simple evolution of the reference state Ω_t given in (11). In

mathematical terms this can be done by the following ansatz

$$\Phi_t^{\approx}(x_1,\dots,x_N) = \sum_{k=1}^{N}(-1)^{k+1}\int d^3y_k\,\eta_t(x_k,y_k)\Omega_t(x_1,\dots,y_k,\dots,x_N). \tag{14}$$

The summation and the factor $(-1)^{k+1}$ are to ensure the antisymmetry of the wave function. For the choice

$$\Omega_{t=0} = \Omega, \qquad \eta_{t=0} = \chi\otimes\varphi_1^*, \tag{15}$$

one obtains $\Phi_{t=0}^{\approx} = \Phi$ due to the orthonormality of the states $\chi,\varphi_1,\dots,\varphi_N$. The excitation is hence encoded by a two-particle wave function $\eta_t(x,y)$. Its x tensor component, which we shall refer to as *electron component*, tracks the evolution of the initial excitation χ. Its y tensor component, which we shall refer to as *hole component*, tracks the evolution of the corresponding state in the reference Ω_t that is missing.

In order to ensure that $\Phi_t^{\approx}$ approximates the actual Φ_t, it turns out that one has to choose

$$i\partial_t\eta_t(x,y) = H_k^0(x_k)\eta_t(x_k,y_k) - \eta_t(x_k,y_k)\overleftarrow{H}_k^0(y_k) - U(x_k-y_k)\eta_t(x_k,y_k), \tag{16}$$

where we set $\overleftarrow{H}^0(y) = i\alpha\cdot\overleftarrow{\nabla}_y + \beta m$ with $\overleftarrow{\nabla}$ denoting the gradient acting to the left. Hence, the excitation η_t evolves according to a two-particle wave equation obeying the free Dirac dispersion in both tensor components and an attractive pair-potential potential – note the signs: the hole component has positive kinetic energy and the U has a different relative sign as compared to (3) –, as if it would describe a quantum two-particles system of identical particles except for the opposite charge. In consequence, against the uniform background of the vacuum state Ω, Φ_t appears as having one negative elementary charge e and one positive one. According to Born's rule, which applies thanks to the discussed mode of approximation, the position distribution of these two charges is given by $|\eta_t(x,y)|^2d^3xd^3y$. In principle, due to the appearance of these two elementary charges in the transition from Ω to Φ_t, also the evolution of all the other tensor components in the sea is affected: their motion does not exactly follow the one of the true vacuum state Ω – a phenomenon referred to as *vacuum polarization*. This in turn also influences the evolution of η_t. However, this effect is not of order e^2 as in U, but of higher order e^4, and thus much smaller. As long as the excitation comprises only few charges (as compared to N), such polarization effects may be neglected in certain regimes.

In sum, we arrive at an economic two-particle description given by (14), (15), (11) and (16) that approximates the actual N-particle dynamics of the initial excitation Φ in (12). As the initial vacuum state essentially does not change over time – cf. (11) – and since due to (6) the vacuum behaves as if there are no electrons, for all practical purposes, one may be inclined to forget about Ω entirely. The vacuum state Ω then simply encodes a state without excitations. A transition from Ω to Φ_t by means of (3), however, requires the introduction of the two-particle wave function η_t. Further excitations require the introduction of further multi-particle excitations and so on. However, as long as the number of excitations is small compared to N, those excitation wave functions are much less complex objects to keep track off than the actual N-particle dynamics of Ψ_t.

To streamline the mathematics according to this idea, it makes sense to formulate the N-particle wave function dynamics (3) not on $\mathcal{H}^{\wedge N}$ but on a space that only keeps track of the varying number of wave function excitations with respect to a reference state such as Ω. For this purpose one introduces the so-called Fock space $\mathcal{F}_\Omega$, which can be defined by employing the so-called creation and annihilation operators. Algebraically the creation operator ψ^* is given

by

$$\psi^*(f)\,\varphi_1 \wedge \varphi_2 \wedge \ldots \varphi_N = f \wedge \varphi_1 \wedge \varphi_2 \wedge \ldots \varphi_N \tag{17}$$

for any choice of $f, \varphi_k \in \mathcal{H}$. The annihilation operator ψ is defined as the corresponding adjoint ψ. Due to the anti-symmetric tensor product $\wedge$ these operators fulfill the anti-commutation relations

$$\begin{aligned} \{\psi(f), \psi^*(g)\} = \langle f, g\rangle, \qquad \{\psi(f), \psi(g)\} = 0 = \{\psi^*(f), \psi^*(g)\} \qquad \text{for all } f, g \in \mathcal{H}, \\ \text{and furthermore,} \quad \psi(\chi)\Omega = 0 = \psi^*(\varphi)\Omega \qquad \text{for all } \chi \in \mathcal{H}^+, \varphi \in \mathcal{H}^-. \end{aligned} \tag{18}$$

The Fock space $\mathcal{F}_\Omega$ can then be defined as the tensor product $\mathcal{F}_\Omega = \mathcal{F}^e_\Omega \otimes \mathcal{F}^h_\Omega$ of the electron Fock space $\mathcal{F}^e_\Omega$ and hole Fock space $\mathcal{F}^h_\Omega$, which are spanned by finite linear combinations of the form $\psi^*(f_1)\ldots\psi^*(f_n)\Omega$ and $\psi(g_1)\ldots\psi(g_n)\Omega$ for $f_k \in \mathcal{H}^+$ and $g_k \in \mathcal{H}^-$, respectively. In this algebraic construction the specific reference state Ω in $\mathcal{H}^{\wedge N}$ is hidden as Ω is encoded by $|\Omega\rangle = 1 \otimes 1$. The independence of $|\Omega\rangle$ of Ω is, however, an illusion, as Ω is encoded in the relations (18) – acknowledging the fact that Ω is the Dirac sea with all states in $\mathcal{H}^-$ occupied. By construction, there is a well-known canonical isomorphism between $\mathcal{H}^{\wedge N}$ and the N-particle sector of $\mathcal{F}^N_\Omega$, that is, the subspace of $\mathcal{F}_\Omega$ spanned by states with equal numbers of electron and hole excitations. It reads

$$\iota : \mathcal{H}^N \to \mathcal{F}^N_\Omega, \quad \iota(f_1 \wedge f_2 \wedge \ldots \wedge f_N) := \psi^*(f_1)\ldots\psi^*(f_N)\psi(\varphi_1)\ldots\psi(\varphi_N)|\Omega\rangle \tag{19}$$

for $f_k \in \mathcal{H}$. Thus, the approximate excitation $\Phi^{\approx}_t$ for (14) is represented in $\mathcal{F}_\Omega$ by

$$|\Phi_t\rangle = \int d^3x \int dy\, \eta_t(x,y)\psi^*(x)\psi(y)|\Omega\rangle. \tag{20}$$

Using these translation rules implied by ι, we can recast the actual N-particle dynamics (3) generated by the Hamiltonian H^N in terms of creation and annihilation operators as

$$\begin{aligned} i\partial_t|\Psi_t\rangle &= \widetilde{H}|\Psi_t\rangle \\ \widetilde{H} &= \int d^3x\, \psi^*(x)\left(H^0_x + V_t(x)\right)\psi(x) - \int d^3x \int d^3y\, \psi^*(x)\psi^*(y)U(x-y)\psi(y)\psi(x). \end{aligned} \tag{21}$$

This expression is the standard second-quantized Hamiltonian of quantum electrodynamics (QED) in Coulomb gauge when neglecting radiation. Note that in $\widetilde{H}$ the creation and annihilation operators appear in pairs, which ensures that for an initial state in the N-particle sector of $\mathcal{F}_\Omega$ the generated dynamics also remains there.

In conclusion, for an initial wave function $\Psi_{t=0} \in \mathcal{H}^{\wedge N}$, there is a unique initial Fock vector $|\Psi_{t=0}\rangle := \iota(\Psi_{t=0})$ such that the time evolutions $(\Psi_t)_{t\in\mathbb{R}}$ on Hilbert space $\mathcal{H}^{\wedge N}$ and $(|\Psi_t\rangle)_{t\in\mathbb{R}}$ on Fock space $\mathcal{F}_\Omega$ generated by the Hamiltonians H^N in (3) and $\widetilde{H}$, respectively, fulfill $|\Psi_t\rangle := \iota(\Psi_t)$ for all times t. In other words, the N-particle formalism is equivalent to the Fock space formalism when restricting to the N-particle sector. While the former represents the dynamics absolutely, the latter represents the dynamics with respect to a reference state Ω. The latter is, as we described above for the case of the two-particle excitation, tailor-made for finding approximations of excitations of Ω.

5 The appearance of particle creations and annihilations

In using the second quantized formalism in section 4 instead of the N-particle formalism in (3), one is naturally led to a mode of language in which excitations are "created" and "annihilated". When an initial reference vacuum state Ω evolves into an excited state like Φ in (12), an

excitation η_t is created, which effectively evolves according to (16). As can be seen from (16), the interaction between the electron and hole components of η_t is attractive, which may lead to a recombination unless an external influence $V(t,x)$ prevents it. In such a process the evolved excitation $\Phi_t^{\approx}$ decays back to the reference state Ω resulting in the annihilation of the excitation.

One may be inclined to refer to the electron component of η_t as describing one created "electron" and the hole component as describing one created "hole", or synonymously, "positron". This, however, is an abuse of the well-defined term "electron" given by the introduced ontology, namely a persistent particle that moves electronwise. Even if the state $\Phi_t^{\approx}$ in (14) were not only an approximation but the actual N-particle wave function, by the form of the velocity law (1) and antisymmetry, the electron component of η_t would not influence only one electron, but all of them. It is therefore the collective motion of all electrons that causes the excitation η_t to appear. It would thus be a misconception to interpret η_t as guiding a two-particle system.

On the mathematical level of wave functions, the hole component of η_t simply encodes what has to be taken out of the reference state Ω, while the electron component of η_t encodes what has to be added to Ω in order to get a good approximation of the actual quantum state Ψ_t of the Dirac sea. This convenient approximation in terms of Ω comes at the price of losing direct information about the actual particle trajectories: as mentioned, in this case only the statistics about particle positions encoded in the wave function are under control. Hence, all that can be said about η_t is that against the uniform distribution of a sea of electrons described by Ω – due to the equilibrium condition (6) –, $|\eta_t(x,y)|^2 d^3x d^3y$ is the probability of finding an additional negative charge and the absence of a negative charge (or mathematically equivalently, the presence of a positive charge) in the volumes d^3x at x and d^3y at y, respectively.

Nevertheless, the excitation η_t has the same properties as wave-packets and would in principle be capable of guiding two particles. To emphasize the difference between such excitations and actual particles, the term *quasi-particles* is usually employed in physics (as, e.g., phonons being the quasi-particles describing quantum excitations of crystal lattice oscillations). In this view, electron and hole components of η_t describe electron and hole quasi-particles having exactly the same properties except for the opposite charge, as can be seen from (16). The bottom line is that at all times there are N electrons in the Dirac sea. None of them was ever created or destroyed. It is their collective motion, which may deviate from the vacuum motion, that gives rise to what we call excitations or electron and hole quasi-particles.

One can emphasize the quasi-particles character of the excitations already in the mathematical formalism. For this it is convenient to split ψ^*, ψ into electron and positron creation and annihilation operators

$$\psi^*(f) = b^*(f) + c^*(f) \quad \text{with} \quad b^*(f) = \psi^*(P^+ f), \quad c^*(f) = \psi^*(P^- f) f \in \mathcal{H}, \tag{22}$$

where $P^\pm$ are the orthogonal projectors on $\mathcal{H}$ to $\mathcal{H}^\pm$, respectively. The number of electrons and hole quasi-particles can then be given by

$$N^e = \sum_n b^*(\chi_n) b(\chi_n), \qquad N^h = \sum_n c(\varphi_n) c^*(\varphi_n), \tag{23}$$

using any orthonormal bases $(\chi_n)_{n=1,\ldots,N}$ in $\mathcal{H}^+$ and $(\varphi_n)_{n=1,\ldots,N}$ in $\mathcal{H}^-$. Note that by virtue of (18), for instance,

$$N^e|\Omega\rangle = 0 = N^h|\Omega\rangle, \tag{24}$$

while for Φ in (12), $N^e|\Phi\rangle = 1 = N^h|\Phi\rangle$ as expected.

However, much caution has to be taken when interpreting the particle number operators in (23). First these operators do not yield a definite number before taking, for instance, their expectation value. Even then, rather than an absolute number, these operators record only the number of excitations relative to the chosen reference state Ω, as can be seen from (24),

i.e., (18). Furthermore, recall that in the preceding section when considering the case of zero external influence – $V(t,x) = 0$ –, we argued that the ground state of the free theory, Ω, is a good candidate to approximate the vacuum. This choice stemmed from the need to find a good approximation and was by no means unique (which is why we regarded a whole class of reference equilibrium states – recall that Ω was just a representative in the case of no external influence $V(t,x) = 0$). Depending on the kind of approximation in mind, one choice of reference state Ω might be better than another. Changing the reference state, however, naturally changes the meaning of the number operators, as excitations are defined with respect to it.

For instance, when an external influence $V(t,x)$ is present, the equilibrium conditions – i.e. fulfilling the conditions (a) and (b) discussed in (6) and below – naturally change. When $V(t,x)$ only changes very slowly, physicists usually employ the so-called Furry picture: for each time t one chooses as reference Ω the state in which all negative energy states of the Hamiltonian $H^0(x) + V(t,x)$ are filled. However, it is shown in Fierz and Scharf (1979) that even this seemingly canonic construction of reference states (23) is not Lorentz invariant – in the sense that what might appear as an empty vacuum in one reference frame may contain many quasi-particles in another. This is because the spectrum, being an energy, transforms like time under Lorentz boosts. A good choice of a reference state will ultimately depend on the detector model, namely on which excitations of the equilibrium conditions (6) cause clicks. But this is an artifact of our approximation only, that is, of the attempt in solving the complexity problem (II) of section 2. The Dirac sea model is independent of observers or detector models: there are always N electrons in the sea moving according to laws (1) and (3).

Consequently, the choice of Ω should rather be seen as a choice of suitable coordinates with respect to which one may track the complicated N-particles dynamics (3) most conveniently. In principle, any choice is possible, since the isomorphism (19) between the N-particle formalism (3) and the formalism in terms of creation and annihilation operators (21) does not depend on particular properties of Ω. The number operators that are defined with respect to this choice Ω, however, can be assigned a physical interpretation only with respect to Ω. For example, if Ω fulfills the equilibrium conditions (a) and (b) discussed in (6) and below, the net sum of all pair potentials between the electrons vanishes and each electron effectively feels a constant potential and, thus, moves freely as if it were in a vacuum (a prime example of dark matter as effectively there is no interaction except for maybe gravitation). The presence of an excitation of Ω disturbs those conditions, resulting in an effective interaction between all electrons in the sea with the electron-hole excitation η_t, which dictates the form of (16). The density $|\eta_t|^2$ then describes the distribution of an additional negative and positive charge with respect to the uniform background distribution of Ω. Only in this sense do the particle number operators record the number of in principle physically measurable excitations. This is what is gained from the introduction of an ontology of persisting particles: it is capable of explaining the import of all the relevant mathematical quantities of the theory.

The Unruh effect (Unruh (1976)) can also be understood in this way: the equilibrium conditions were chosen to find a good approximation of the complex N-particle dynamics and are at best Lorentz invariant. Switching to an accelerated frame will in general violate those conditions. Consequently, states Ω and Ω' describing a vacuum in a rest frame and an accelerated frame, respectively, may differ. Thus, Ω may appear in the accelerated frame as excitation of Ω' possibly involving many electron-hole quasi-particle pairs. In sum, there is no point in giving an ontological significance to these quasi-particles – which are the ontological particles in the proposal of Bell (1986) –, since there is no canonical choice for Ω.

6 Conclusion: The merits of the Bohmian approach

Before drawing our conclusion, three points are worth being addressed. In the first place, one may gain the impression that the introduced formalism based on a reference state Ω, which fills

the spectral subspace $\mathcal{H}^-$ with actual particles that move electronwise and leaves $\mathcal{H}^+$ empty, violates one of the much celebrated symmetries of QED, namely the so-called charge symmetry (a symmetry that is broken in the weak interaction). This is a misconception. Charge symmetry simply demands that the equations of motion remain invariant when the signs of the elementary charge e and the one of the external influence are inverted, which is the case (see (3) and also (16) for the effective equation).

Furthermore, the model that we have discussed is based on a finite, though large, number of electrons. Whenever one or both of the parameters Γ and Λ – the volume of the universe and the cut-off of the interaction – are sent to infinity, the construction of the ground state in (9) would imply $N \to \infty$, as then $\mathcal{H}^-$ comprises countable but infinitely many states. Sending Γ to infinity could on the one hand be seen as carrying out a thermodynamic limit of the finite system. On the other hand, even keeping the volume Γ fixed while sending the momentum cut-off Γ to infinity would as well require $N \to \infty$. In the latter case, the only way to maintain a sensible ontology of particles is to assume that the Dirac sea is not entirely filled. For instance, (9) may be composed only of the first N states in $\mathcal{H}^-$ occupied by electrons. When coupling such a state to an open system such as the photon field, the electron in the Dirac sea may decay to lower and lower energy states that are unoccupied. This would result in a never ending energy transfer from the electron into the photon degrees of freedom that transport this energy to spatial infinity so that the dynamics becomes unstable. Such a situation would of course be unphysical. However, it is due to the unphysical artifact of the photon field acting as an infinite energy sink. In a realistic model of the universe such an energy sink has to be ruled out. To achieve this, the equations of motion of the photon field would have to be supplied with an extra condition ensuring that all emitted photons will at some later time be absorbed again by fermions (see Wheeler and Feynman (1945)) so that no radiation can escape to spatial infinity. For the moment, however, an ontological discussion taking into account the removal of the cut-offs as well has to wait until modern QFT succeeds in finding a well-defined, relativistically interacting equation of motion that will replace equation (3).

Finally, it has to emphasized that even in the non-interacting case, while the statistics about the particle positions (which are encoded in the wave functions of the excitations thanks to Born's law) are Lorentz invariant, the actual particle trajectories generated by the velocity law (1) are not. It is a well-known fact that the latter depend on a preferred foliation of Minkowski space-time – in the setup above, we have chosen equal time hypersurfaces. This circumstance is not an artifact of our model; it is generic to all relativistic versions of BM. The reason is the manifestly non-local nature of QM as demonstrated, for instance, by Bell's theorem and the subsequent experiments. Instead of adding this extra structure of a preferred foliation by hand, it is possible to introduce it in a Lorentz-covariant manner by taking it to be determined by the N-particle wave function itself, or equivalently the Fock space vector (see Dürr et al. (2013b)). We did not do so because this would have made the definition of the Dirac sea model and our arguments for a particle ontology unnecessarily opaque. Furthermore, the actual trajectories of individual electrons figure only in the ontology of our theory, while, due to problem (I) mentioned in section 2, they alone do not allow to infer predictions. It is rather the collective motion of all particles in the Dirac sea that generates the correct position statistics, which are encoded in the excitation wave functions by Born's law; the latter are accessible and Lorentz-invariant.

Despite the still open desiderata, we have demonstrated that the predictions of the SM in the electron sector with cut-offs and neglect of radiation arise naturally from a theory of N persistent particles. Creation and annihilation appear only in an effective description with respect to a chosen reference state. This programme is not only applicable to the electron, but to all fermion sectors.

In conclusion, let us stress again the parallelism with BM for QM: in both cases, the ontology is exactly the same – a fixed, finite number of permanent point particles that are characterized only by their positions – , and the dynamics is of the same type, that is, a deterministic law

describing the motion of the particles on continuous trajectories without any jumps. In both cases, the objective is to answer the questions of what there is in nature on the fundamental level (i.e. permanent particles) and how what there is evolves (i.e. provide a dynamics for the individual processes in nature). From this ontology and dynamics then follow the formalisms of statistical predictions of both standard QM and QFT. In this sense, these predictions are explained by this underlying ontology and dynamics.

The same goes for the solution to the measurement problem that hits QFT in the same way as QM (see Barrett (2014)): the Bohmian point particles are admitted because they explain the measurement outcomes. For instance, the dots on the screen as recorded in the double slit experiment are made up by these point particles. However, this explanation is not achieved by the (primitive) ontology of particles alone, but by this ontology together with the dynamics (i.e. the guiding equation): it is the dynamics that provides for the stability of macroscopic objects, including pointer positions and dots on a screen. As Dickson (2000) convincingly argued, to obtain this stability, no properties of the particles over and above their position in space are needed, but a dynamics is that provides for trajectories such that there are stable macroscopic particle configurations.

Bearing these facts in mind, the solution to the measurement problem provided by Dirac sea Bohmian QFT is of the same type: here again, the Bohmian point particles are admitted because they explain the measurement outcomes. There always is a definite, finite number of particles moving on continuous trajectories, with some of these particles making up the macroscopic phenomena that we see. As discussed, we only have to be more careful to relate those particles and their dynamics to what we see with the naked eye, e.g., the trajectories recorded in cloud chamber detectors. The latter are generated by deviations from the collective motion of the Bohmian particles in an equilibrium state such as the vacuum. Thus, to stress again, what explains these phenomena is not the mere fact of there being particle configurations, but the particle dynamics: it consists in this case of excitations from a sea of particles in an equilibrium state, with these excitations, as elaborated on above, implying a change of the motion of in principle all the particles in the sea. To sum up, what explains the traces in the cloud chamber are these excitations as affecting the motion of the particles in the sea.

Hence, whatever may eventually be the fate of the Bohmian approach, it works for QFT in the same way as for QM. Thus, if this approach is a serious contender for the ontology of QM, so it is a serious contender for the ontology of QFT.

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